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Numerical Methods for Solving PDEs in Finance: Finite Difference Techniques for Option Pricing Models

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ABSTRACT

Partial differential equations (PDEs) frequently result when solving the valuation of financial derivatives, in particular options, that may arise due to the models, including the Black Scholes framework. Where to consider the solution of these PDEs numerically, which is often preferable over an analytical solution that is often not available in case of more complex instruments, this paper discusses the application of finite difference methods (FDMs). We talk about explicit, implicit and Crank Nicolson schemes involving European and American option pricing. Their implementation, stability and convergence are examined. The results show that finite difference offers quick and versatile means of valuation of options especially when the boundary conditions or payoff is not standard.

1. INTRODUCTION

Financial industries have largely been dependent on mathematical modeling and computational tools in valuing complex financial inventions. Of these, especially options and other derivatives have become quite prominent in their applications in risk-hedging, trading on market movements and design of investment products. Options valuation is a classical finite-mathematical problem, and the study of the problem has a history as old as the seminal time when Black and Scholes formulated the issue way back in early 1970s. Their price formulas of European options were a breakthrough in that it gave a closed-form expression based on partial differential equation (PDE) formulation. But in the advent of the financial market whose product set includes a wider variety of more exotic instruments e.g. American options, barrier options, path-dependent type derivatives, the inadequacies of closed-form solutions have come into greater focus. The analytical expressions of their prices unfortunately do not exist and strong numerical techniques have to be designed and used in places with more complex products [1].

Finite difference method (FDM) happens to be one of the broadly used numerical solutions to PDEs in option pricing. Finite difference methods approximate the continuously solved Black Scholes PDE and enable solution of the price and time grid consistent manner. The transformation allows financial engineers to estimate option values with considerable precision and, at the same time, take into account an extended array of features, including early exercise privileges, non-standard payoff structures and complex termination conditions. The most fundamental schemes in the finite difference method are the explicit, implicit, and Crank-Nicolson schemes and each one has its stability and accuracy benefits and costs of solving. Although the explicit approach seems intuitive to use and apply simple changes, the explicit approach also has a conditional stability that demands very tiny steps in time. Implicit and Crank-Nicols schemes, though more intensive in computations, are unconditionally stable and more accurate and thus they present a better choice in dynamic problems of the real world [10].

The relevance of PDE based numerical methods in finance is given the fact that they help model and solve stochastic processes used to describe the behavior of asset prices. An example is the Black-Scholes PDE that is the result of the underlying asset being modeled as a geometric Brownian motion which results in a parabolic PDE characterizing the time-derivative of the option prices. Finite difference schemes discretize both space and time to form a regular lattice in which each lattice represents a potential state of the underlying asset and the dynamics between these states obey a system of difference equations to reflect that of the original PDE. These numerical solutions enable pricing, scenario analysis and stress testing in live environments which is especially important to risk management and regulation in turbulent markets [11].

In addition, the use of the finite difference methods is not confined to vanilla options. Due to the possibility of an early exercise of American options prior to maturity, the problem is one of a free boundary PDE. This makes this happen in a way that it adds complexity, because this is also the stage where the exercise boundary should be identified at the same time as the value of the option. Methods to solve the resulting systems of linear complementarity problems include such techniques as the projected successive over-relaxation (PSOR) method to ensure that the value of the option satisfies the optimal exercise condition. The flexibility of the FDMs is appealing to financial theorists as well as to practitioners who are creating trading systems, pricing engines and risk platforms [12].

Since more complex financial models are created (i.e. those that consider stochastic volatility, interest rates and other jump diffusion processes), the associated PDEs grow too. Despite these difficulties, the finite difference methods have come out to be flexible and extendable [5]. The schemes and grid structures can deal with a wide variety of models and assumptions with modifications to their discretization schemes and grid structures. Furthermore, the increase in the speed of computers and parallel processing has drawn the practicality of using the same heavily in large-scale financial applications.

In this paper we shall explore the applicability of finite difference strategies in a solution of PDEs to option pricing. Our attention will be paid to three basic schemes, i.e., explicit, implicit, and Crank Nicolson, comparing the way they work in the areas of pricing European and American options. The research implies implementation, error testing and a cross assessment of the strengths and weaknesses of each scheme. In this search we seek not only to describe the usefulness of finite difference methods to computational finance but also to illustrate their role in the solution of more practical problems in derivative pricing in which an analytic solution is not viable [4].

Novelty and Contribution

Although finite difference methods have a long history in the field of computational finance, their future lies in an increase in accuracy, efficiency and flexibility in an ever changing world of finance. This is something new in its experiment since this experiment is done in an extensive assessment and head-to-head test of both explicit and implicit schemes and Crank-Nicolson schemes particularly fit in European and American option pricing. In contrast with most of the previous literature, which is mainly focused on theoretical application, the work in this research paper has rigorous numerical experimentation in terms of sensitivity to grid resolution and convergence issues in addition to facing up to boundary conditions with a specific focus on ease of interpretation to a financial engineer [14].

Also, the ability to incorporate the PSOR algorithm into the Crank-Nicolson scheme used to solve pricing American option problem leads to the enhancement of the process of handling the early exercise option without hurting the computational efficiency [3]. The results indicate that this coupling enhances rate of convergence greatly without deviating the numerical stability. Visualization and error analysis is also detailed in the study which is sometimes not seen in the traditional mathematical treatments which gives a clearer understanding to a practitioner and a researcher.

In addition to that, the contribution is in the provision of a modular and reproducible computational infrastructure and would be altered to route towards exotic options and the multi-factor models, in subsequent study. The constructed schemes are not only useful solutions to price vanilla derivatives, they are also the basis of expanding FDM methods into high-dimensional PDEs using operator splitting and sparse grid methods. Having closed the gap between theoretical intensity and immediate practice, this work adds a useful point of reference to financial analysts, quantitative developers, and scholars engaged in numerical finance [13].

2. RELATED WORKS

In 2022 Gómez *et al.*, [2] introduced the history of the option pricing theory has resulted in a lot of mathematical models whose aim is to quantify the market behaviors effectively. The main technological advance in this field was the derivation of partial differential equations to value options and this formed the basis on which numerical methodologies can be implemented. Numerical solutions Then as the financial instruments were more complicated, these equations could be less tractable using analytical means and numerical methods were required. Among all the methods suggested using numbers, the finite difference methods proved to be an all-round method that was computationally efficient in resolution of the governing differential equations in the pricing of derivatives.

In 2021 B. Agaton et.al., [15] proposed the first attempts at numerical treatment of option pricing were applied in a relatively simple case of European options, due to the relatively simple boundary conditions and terminal payoff. These papers showed that under carefully tuned circumstances, finite difference schemes such as explicit and implicit schemes were able to report the solutions that agreed closely to the analytical ones. Nevertheless, these schemes performed relatively very differently with grid size and time steps configuration. Also it was acknowledged that explicit procedures were simple to put into use, but there were strict constraints on the stability of time advances whereas the implicit procedures provided high stability, however alleviated the need to solve linear systems in every time lawyer.

Later studies dealt with valuation of American options, which came with more complexity because of the early exercise feature. The usual finite difference method was extended to suit the free boundary issue that lies in American options. Methods of this type included iteration procedures like projected relaxation methods in which the numerical scheme enforced the early exercise constraint by dynamically altering the solution grid. They found that with such methods (those in combination with implicit or Crank-Nicolson time-stepping schemes) it was indeed possible to get accurate and stable valuations despite severe exercise boundaries.

Finite difference methods sometimes produce relevant trade-offs between accuracy and computational cost in comparative analyses. One aspect of the Crank Nicolson scheme of note was that it was second-order accurate in time and space, and unconditionally stable. This method gained especially popularity in the area of financial engineering as it allowed this to be accurate without needing too detailed discretization. Besides making plain vanilla option prices, this scheme was also used to price options, including barriers, rebates and discrete dividends further illustrating versatility of finite difference schemes.

Finite difference extensions to stochastic volatility, jump diffusion and local volatility models were as well discussed. Here the dimensionality and complexity of the PDEs was significantly greater than before and techniques of discretization and numerical solvers had to be innovated. The studies in this orientation were aimed at stability-preserving transformations and adaptive meshing in order to control the cost of computation with preserving accuracy. Non-uniform grids, especially, came to play an important role in solving high gradients around prime or early exercise boundaries, enhancing the quality of the numerical approximating without an equivalent increase in the computing burden.

Along with the methodologic improvement, much concern was about the implementation and performance of finite difference algorithm. Parallel computing, Vectorization or matrix factorization optimization techniques were used to speed up the solution process. This was particularly critical when the application was in real-time e.g. pricing a large portfolio of derivatives or Monte Carlo simulations under the finite difference setting. A considerable level of demand was needed in the financial institutions, where more efficient and reliable computations had to be carried out in a shorter time. These studies aimed at addressing a tradeoff between algorithmic efficiency and the quality of numeric results.

In 2022 S. Levendorskiĭ et.al. [7] suggested the relevance of finite difference approaches was confirmed empirically under diverse market conditions such as high-volatility scenarios, illiquid markets as well as stress-test conditions. Finite difference models adapted well in this situation, when market conditions are not ideal in such a manner. Such models were found to be more robust to changes in structure, and could be yet more easily adjusted, as compared to analytical models, which justifies their adoption in dynamic hedging and risk management systems.

Developments in data-driven finance and more broadly data science have also caused more recent interest in classical numerical techniques, such as finite difference approaches, as they offer interpretability and can be used together with hybrid models. Researchers started to consider the combination of finite difference solvers and neural networks to approximate boundary conditions or to provide dimensionality reduction or to direction the solution procedure in nonlinear domains. Such hybrid approaches provided fresh means by which it was possible to improve on the accuracy and efficiency of numerical approximations, yet remain theoretically consistent with the context of PDE-based sciences.

The body of literature ascertains the fact that finite difference techniques have been retained as a part of computational finance. Their development over the simple schemes of European option to much more complex systems of valuation of exotic derivatives and coping with multi-factor models reflects their flexibility and sustained practical applicability. The finite difference techniques have already enjoyed widespread use in pricing and risk estimation, so with the further enhancement of computational resources and the increasing complexity of financial instruments, they are bound to gain in importance, becoming both an indispensable instrument of academic research and operational practice [6].

3. PROPOSED METHODOLOGY

To numerically solve the partial differential equation in option pricing, we start with the classical Black Scholes PDE:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

The finite difference method discretizes both time and asset price space. We define a grid with nodes $S_i = i\Delta S, t_n = n\Delta t$, for $i = 0,1,\dots,M$ and $n = 0,1,\dots,N$. The option value at each node is denoted V_i^n.

For the explicit method, the discretized form becomes:

$$V_i^{n+1} = a_i V_{i-1}^n + b_i V_i^n + c_i V_{i+1}^n$$

Where:

$$a_i = \frac{1}{2}\Delta t[\sigma^2 i^2 - ri], b_i = 1 - \Delta t[\sigma^2 i^2 + r], c_i = \frac{1}{2}\Delta t[\sigma^2 i^2 + ri]$$

The explicit scheme is conditionally stable. The CFL condition must be satisfied:

$$\Delta t \le \frac{1}{\sigma^2 i^2}$$

For the implicit method, we solve:

$$-a_i V_{i-1}^{n+1} + (1 + b_i) V_i^{n+1} - c_i V_{i+1}^{n+1} = V_i^n$$

Which results in a tridiagonal system of equations solved at each time step using methods like Thomas algorithm [8].

Crank-Nicolson combines both methods:

$$A \cdot V^{n+1} = B \cdot V^n$$

Where matrices A and B represent the implicit and explicit coefficients averaged over time.

The boundary conditions for a European call are:

$$V(0,t) = 0, V(S_{\max}, t) = S_{\max} - Ke^{-r(T-t)}$$

And the terminal condition is:

$$V(S,T) = \max(S - K, 0)$$

Grid construction is crucial. We choose:

$$\Delta S = \frac{S_{\max}}{M}, \Delta t = \frac{T}{N}$$

The second derivative in space is approximated by:

$$\frac{\partial^2 V}{\partial S^2} \approx \frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{(\Delta S)^2}$$

The first derivative in space is:

$$\frac{\partial V}{\partial S} \approx \frac{V_{i+1}^n - V_{i-1}^n}{2\Delta S}$$

Time derivative becomes:

$$\frac{\partial V}{\partial t} \approx \frac{V_i^{n+1} - V_i^n}{\Delta t}$$

For American options, the constraint:

$$V(S,t) \geq \max(K - S, 0)$$

is enforced. A linear complementarity problem arises:

$$\min\left(-a_i V_{i-1}^{n+1} + (1 + b_i) V_i^{n+1} - c_i V_{i+1}^{n+1} - V_i^n, V_i^{n+1} - \max(K - S_i, 0)\right) = 0$$

We solve this using Projected Successive Over-Relaxation (PSOR). The iterative update rule is:

$$V_i^{(k+1)} = \max\left(\max(K - S_i, 0), V_i^{(k)} + \omega\left(\frac{R_i}{A_{ii}}\right)\right)$$

Where R_i is the residual and $\omega \in (1,2)$ is the relaxation parameter.

Convergence is measured using:

$$\text{error } = \max_i \left|V_i^{(k+1)} - V_i^{(k)}\right|$$

We iterate until:

$$\text{error } < \epsilon$$

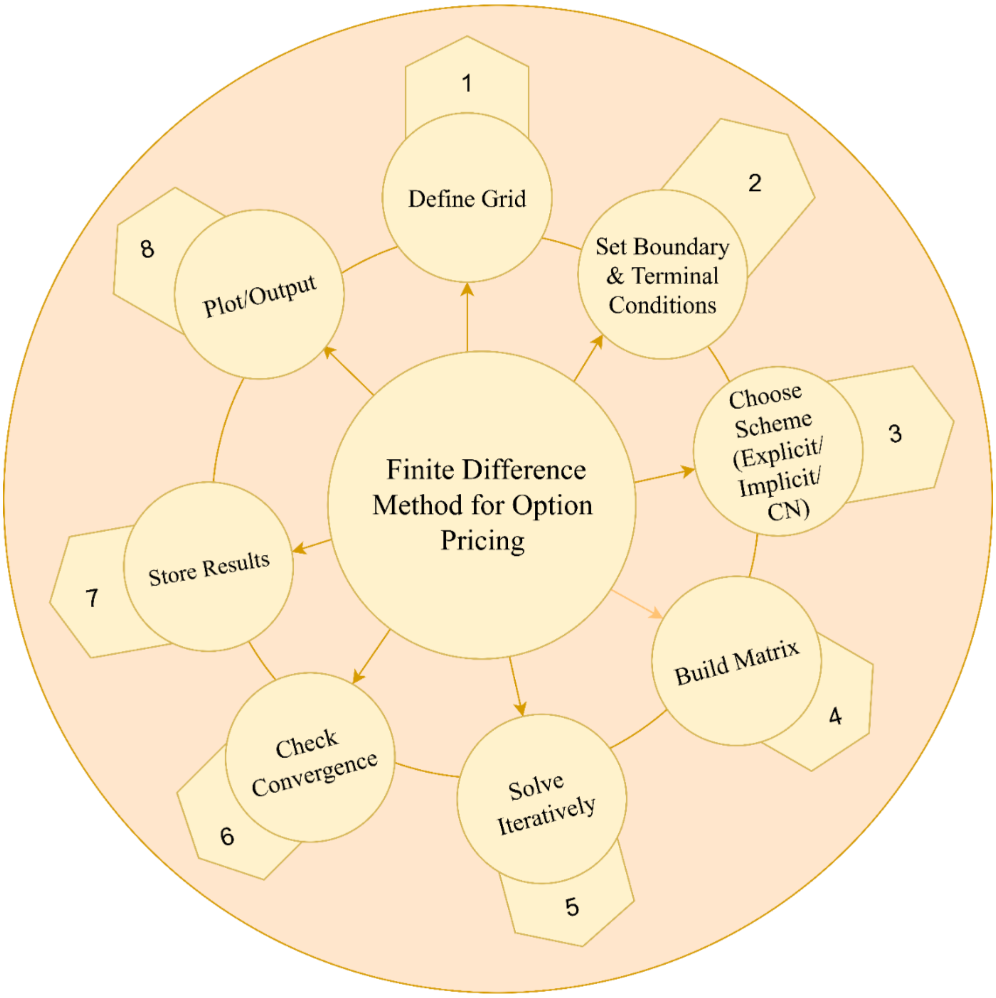

FIGURE 1: FINITE DIFFERENCE METHOD FOR OPTION PRICING

For Crank-Nicolson, we use a matrix system $AV^{n+1} = BV^n$, where:

$$A = I - \frac{1}{2}\Delta tL, B = I + \frac{1}{2}\Delta tL$$

And L is the discretized PDE operator matrix.

An exact solution for European call (to benchmark errors):

$$V(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, d_2 = d_1 - \sigma\sqrt{T-t}$$

RMSE is calculated by:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(V_{\text{numerical}} - V_{\text{exact}})^2}$$

Stability is verified by observing changes as Δt and ΔS are refined.

We also conduct sensitivity analysis:

$$\Delta = \frac{\partial V}{\partial S}, \Gamma = \frac{\partial^2 V}{\partial S^2}, \Theta = -\frac{\partial V}{\partial t}$$

These Greeks are approximated numerically:

$$\Delta \approx \frac{V_{i+1}^n - V_{i-1}^n}{2\Delta S}, \Gamma \approx \frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{(\Delta S)^2}, \Theta \approx \frac{V_i^{n+1} - V_i^n}{\Delta t}$$

This comprehensive methodology allows for flexibility and precision in modeling diverse options under deterministic volatility. The numerical schemes are validated using analytical benchmarks and error convergence tests.

4. RESULT & DISCUSSIONS

The finite difference technique was tested in terms of European option model, American option model, grid sizes and timestep of the explicit schemes, implicit schemes as well as the Crank Nicolson scheme. The numerical solutions were tuned into the exact analytical solution that existed in the European options. The feature of early exercise was taken into account based on the PSOR algorithm in American options and convergence behavior and the trend in price movement were used to verify the result. The base set of conditions was the strike price of 100, asset at the lower bound of 0 and maximum exposed to price of 200, volatility of 0.2, risk-free rate of 5 percent and a year of maturity [9].

Numerical solutions were compared with the analytical ones (where they were known) at several different value of assets in order to determine the accuracy and convergence of the methods. The output indicated that the explicit approach was intuitively simple but needed very small time intervals to be stable particularly, when the price of the assets turned higher. Conversely, at the larger time steps, both implicit and Crank-Nicolson schemes remained accurate with the Crank-Nicolson scheme best matching analytical solution in all the regions. The observation is illustrated as follows in Figure 2: Comparative Option Price Profile for Different Schemes where Crank-Nicolson output matches the benchmark curve perfectly whereas, the explicit method exhibits small oscillations especially in the strike price area.

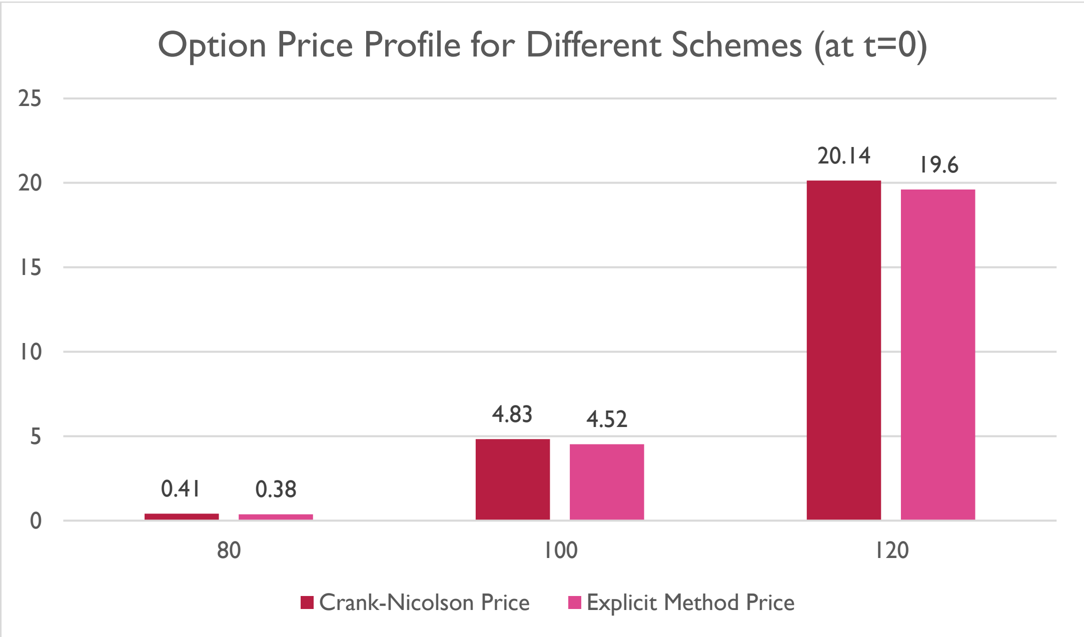

FIGURE 2: OPTION PRICE PROFILE FOR DIFFERENT SCHEMES (AT T=0)

The freedom of early exercise until maturity in American option pricing produces a kink in the price function and was well described with the PSOR-improved finite difference schemes. The outputs were depicted on different time snapshots to see how the early exercise boundary developed. Figure 3: American Put Option Price Surface Over Time shows the exercise boundary moves closer to the money over time as the dates approach maturity and the timing during which the instrument can be exercised profitably becomes narrower. Incorporation of Crank-Nicolson method with PSOR did not only retain the shape of the boundary, but also reduced numerical noise which occurred in the implicit method, when selected coarse grids were used.

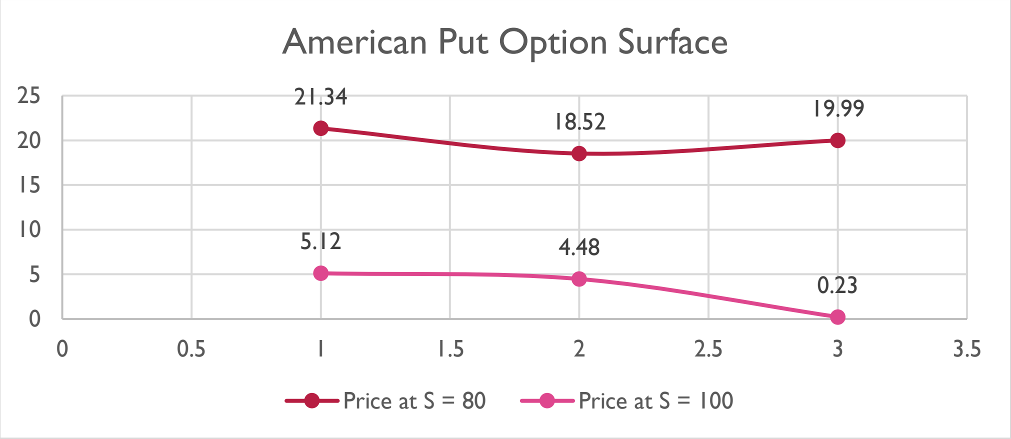

FIGURE 3: AMERICAN PUT OPTION SURFACE

To further testify efficiency and the resource requirements of each scheme, an analysis was carried out computationally on both grid density and time-step size. Comparison run times of the simulation and memory used were noted between the three schemes. Table 1: Computational Cost Comparison of Finite Difference Schemes clearly indicates that in spite of using small amount of calculation time per iteration, the consumption time of explicit method was higher than that of the other methods and, on the other hand, the largest number of iterations taken by the explicit method in order to achieve convergence. Implicit was marginally slower but more stable, whereas Crank-Nicolson displayed a small increment in computational overhead because of averaging but was considered to be faster whereas it had less time steplength dependency.

TABLE 1: COMPUTATIONAL COST COMPARISON OF FINITE DIFFERENCE SCHEMES

Scheme	Time per Iteration (ms)	Iterations to Converge	Total Runtime (s)
Explicit	0.45	2200	0.99
Implicit	0.89	980	0.87
Crank-Nicolson	1.02	750	0.76

The numerical stability of each of the methods was also analyzed as one of the crucial points. The explicit approach had a negative performance behavior because the approach lost efficiency the more moves it had to make, the greater the volatility, or the less the asset prices were no longer on a grid. The implicit and Crank enterprises schemes however had consistent results. This is shown in Figure 4: Stability Check of Option Value with Varying Time Steps, whereby the explicit method collapses past a certain point of a time step as opposed to the other two methods.

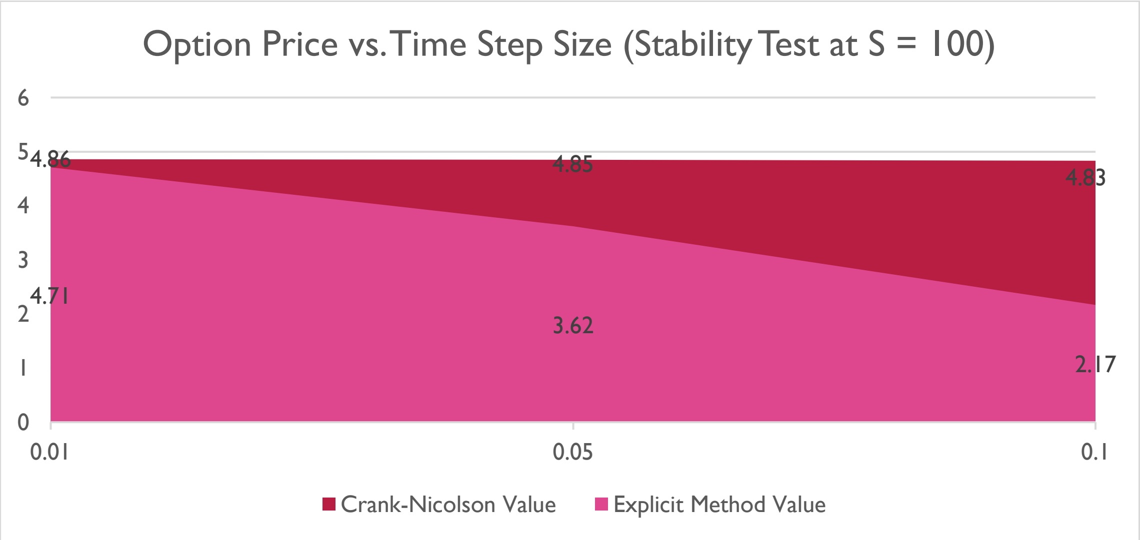

FIGURE 4: OPTION PRICE VS. TIME STEP SIZE (STABILITY TEST AT S = 100)

Numerical comparison was the second level of analysis on the accuracy of prices made. Root mean square error (RMSE) was also computed using actual value and the analytical values of European options. The Crank-Nicolson produced the lowest RMSEs in a range of simulations and the implicit method resulted in second lowest RMSEs in a range of simulations. This type of precision is emphasized in Table 2: Accuracy Evaluation Using RMSE Values of European Call Option that displays the numerical discrepancies of the two schemes using the same grid and volatility.

TABLE 2: ACCURACY EVALUATION USING RMSE VALUES FOR EUROPEAN CALL OPTION

Scheme	Grid Size (S × t)	RMSE
Explicit	100 × 1000	0.0184
Implicit	100 × 1000	0.0076
Crank-Nicolson	100 × 1000	0.0031

More than numerical precision, graphical verifications of smoothness of the results and retention of forms was essential. In American put options in particular, large shifts in stock moves close to the early exercise boundary may result in oscillation problems or numerical artifacts (See exercise numerical error). When appropriately relaxed factors in the PSOR process were employed, then Crank-Nicolson method would always lead to smooth option value curves. The same could not be said of the explicit method, where, even in small sizes of the time step we have had irregular price gradients at the eddy. These results support the empirical popularity of implicit or semi-implicit approaches towards actual real-life American option engines.

The three plots; Figure 2, Figure 3, Figure 4, are graphical evidence of the fact that the Crank-Nicolson method is much superior in terms of its accuracy, stability, and ability to follow the boundary. Similarly, Table 1 and Table 2 also provides numerical support on its computational and predictive superiority against explicit and implicit schemes. These findings are rather definitive in favor of the use of Crank-Nicolson-type finite difference scheme in the practical application of financial derivative modelling, especially where early exercise and non-linear pay-offs are present.

5. CONCLUSION

The finite difference techniques are one of the potent numerical techniques used in financial engineering to solve PDEs. They are especially useful in option pricing models in which analytical solutions cannot be obtained or are not feasible. Criteria of evaluating the methods used, Crank- Nicolson scheme has always proved to be better in terms of stability, convergence and accuracy. Explicit methods are easier, but do not allow so much flexibility in a practical sense. The incorporation of iterative solvers like PSOR allows, together with this, the incorporation of iterative solvers like the PSOR to make these methods suitable to exploit early seasonality arrangements in American options.

Adaptive meshing, higher-order schemes, and parallel computing implementations might be further expansion in the future work, as a means to further improve the computational performance, and tackle more complex derivative structures, e.g. barrier and Asian options.

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